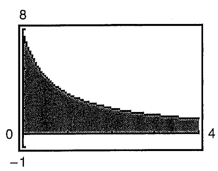
(b) $\int x^n \ln x\, dx = x^{n+1} \ln x/(n+1) - x^{n+1}/(n+1)^2 + C$

75. False. Substitutions may first have to be made to rewrite the integral in a form that appears in the table.

77. $32\pi^2$ **79.** 1919.145 ft-lb

81. (a) $V = 80 \ln(\sqrt{10} + 3) \approx 145.5 \text{ ft}^3$
$W = 11{,}840 \ln(\sqrt{10} + 3) \approx 21{,}530.4 \text{ lb}$
(b) (0, 1.19)

83. (a) $k = 30/\ln 7 \approx 15.42$
(b)



85. Putnam Problem A3, 1980

Section 8.7 (page 576)

1.

x	−0.1	−0.01	−0.001	0.001	0.01	0.1
$f(x)$	1.3177	1.3332	1.3333	1.3333	1.3332	1.3177

$\frac{4}{3}$

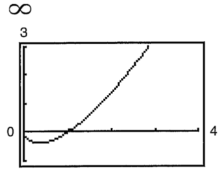
3.

x	1	10	10^2	10^3	10^4	10^5
$f(x)$	0.9900	90,483.7	3.7×10^9	4.5×10^{10}	0	0

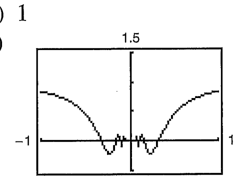
0

5. $\frac{3}{8}$ **7.** $\frac{1}{8}$ **9.** $\frac{5}{3}$ **11.** 4 **13.** 0 **15.** 2
17. ∞ **19.** $\frac{11}{4}$ **21.** $\frac{3}{5}$ **23.** 1 **25.** $\frac{5}{4}$ **27.** ∞
29. 0 **31.** 1 **33.** 0 **35.** 0 **37.** ∞
39. $\frac{5}{9}$ **41.** 1 **43.** ∞

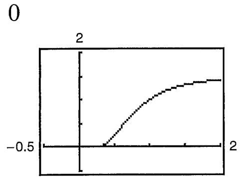
45. (a) Not indeterminate
(b) ∞
(c)



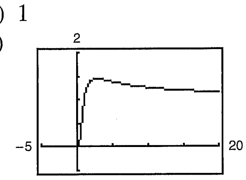
47. (a) $0 \cdot \infty$
(b) 1
(c)



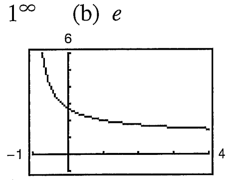
49. (a) Not indeterminate
(b) 0
(c)



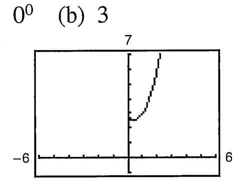
51. (a) ∞^0
(b) 1
(c)



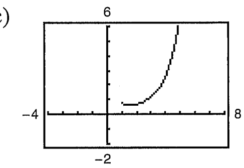
53. (a) 1^∞ (b) e
(c)



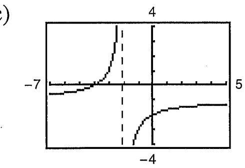
55. (a) 0^0 (b) 3
(c)



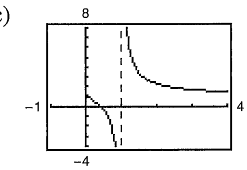
57. (a) 0^0 (b) 1
(c)



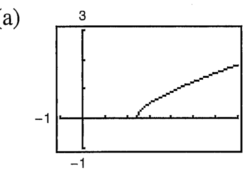
59. (a) $\infty - \infty$ (b) $-\frac{3}{2}$
(c)



61. (a) $\infty - \infty$ (b) ∞
(c)

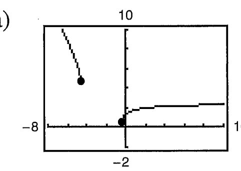


63. (a)



(b) $\frac{1}{2}$

65. (a)



(b) $\frac{5}{2}$

67. $\frac{0}{0}, \frac{\infty}{\infty}, 0 \cdot \infty, 1^\infty, 0^0, \infty - \infty$

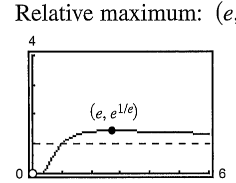
69. Answers will vary. Examples:
(a) $f(x) = x^2 - 25,\ g(x) = x - 5$
(b) $f(x) = (x - 5)^2,\ g(x) = x^2 - 25$
(c) $f(x) = x^2 - 25,\ g(x) = (x - 5)^3$

71.

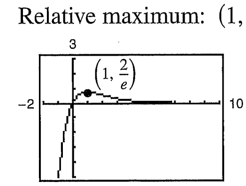
x	10	10^2	10^4	10^6	10^8	10^{10}
$\frac{(\ln x)^4}{x}$	2.811	4.498	0.720	0.036	0.001	0.000

73. 0 **75.** 0 **77.** 0

79. Horizontal asymptote: $y = 1$
Relative maximum: $(e, e^{1/e})$

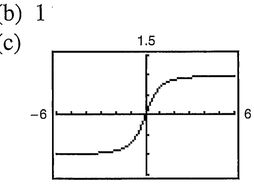


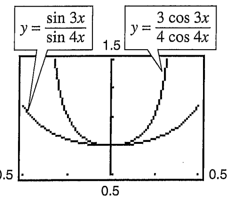
81. Horizontal asymptote: $y = 0$
Relative maximum: $(1, 2/e)$



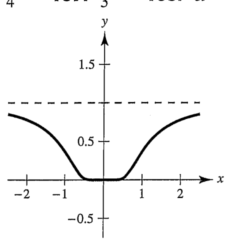
83. Limit is not of the form 0/0 or ∞/∞.
85. Limit is not of the form 0/0 or ∞/∞.
87. Limit is not of the form 0/0 or ∞/∞.

89. (a) $\lim_{x\to\infty} \frac{x}{\sqrt{x^2+1}} = \lim_{x\to\infty} \frac{\sqrt{x^2+1}}{x} = \lim_{x\to\infty} \frac{x}{\sqrt{x^2+1}}$
Applying L'Hôpital's Rule twice results in the original limit, so L'Hôpital's Rule fails.
(b) 1
(c)

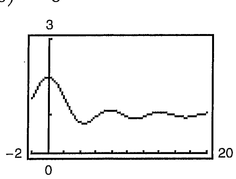


91. 

As $x \to 0$, the graphs get closer together (they both approach 0.75). By L'Hôpital's Rule,
$$\lim_{x\to 0} \frac{\sin 3x}{\sin 4x} = \lim_{x\to 0} \frac{3 \cos 3x}{4 \cos 4x} = \frac{3}{4}.$$

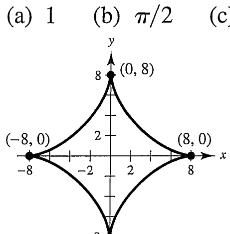
93. $v = 32t + v_0$ **95.** Proof **97.** $c = \frac{2}{3}$ **99.** $c = \pi/4$
101. False: L'Hôpital's Rule does not apply, because $\lim_{x\to 0}(x^2 + x + 1) \neq 0$. **103.** True
105. $\frac{3}{4}$ **107.** $\frac{4}{3}$ **109.** $a = 1, b = \pm 2$ **111.** Proof
113. 

$g'(0) = 0$

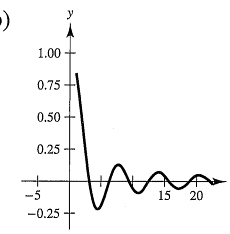
115. (a) $0 \cdot \infty$ (b) 0
117. Proof
119. (a)–(c) 2
121. (a) 

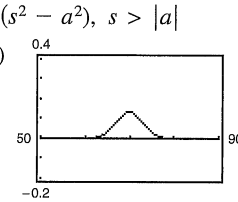
(b) $\lim_{x\to\infty} h(x) = 1$
(c) No
123. Putnam Problem A1, 1956

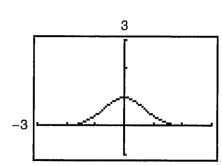
Section 8.8 (page 587)

1. Improper; $0 \leq \frac{3}{5} \leq 1$ **3.** Not improper; continuous on $[0, 1]$
5. Not improper; continuous on $[0, 2]$
7. Improper; infinite limits of integration
9. Infinite discontinuity at $x = 0$; 4
11. Infinite discontinuity at $x = 1$; diverges
13. Infinite limit of integration; $\frac{1}{4}$
15. Infinite discontinuity at $x = 0$; diverges
17. Infinite limit of integration; converges to 1 **19.** $\frac{1}{2}$
21. Diverges **23.** Diverges **25.** 2 **27.** $\frac{1}{2}$
29. $1/[2(\ln 4)^2]$ **31.** π **33.** $\pi/4$ **35.** Diverges
37. Diverges **39.** 6 **41.** $-\frac{1}{4}$ **43.** Diverges **45.** $\pi/3$
47. $\ln(2 + \sqrt{3})$ **49.** 0 **51.** $\pi/6$ **53.** $2\pi\sqrt{6}/3$
55. $p > 1$ **57.** Proof **59.** Diverges **61.** Converges
63. Converges **65.** Diverges **67.** Diverges **69.** Converges
71. An integral with infinite integration limits, an integral with an infinite discontinuity at or between the integration limits
73. The improper integral diverges. **75.** e **77.** π
79. (a) 1 (b) $\pi/2$ (c) 2π
81. 

Perimeter = 48

83. $8\pi^2$ **85.** (a) $W = 20{,}000$ mile-tons (b) 4000 mi
87. (a) Proof (b) $P = 43.53\%$ (c) $E(x) = 7$
89. (a) \$757,992.41 (b) \$837,995.15 (c) \$1,066,666.67
91. $P = \left[2\pi NI\left(\sqrt{r^2 + c^2} - c\right)\right]/\left(kr\sqrt{r^2 + c^2}\right)$
93. False. Let $f(x) = 1/(x + 1)$. **95.** True
97. (a) and (b) Proofs
(c) The definition of the improper integral $\int_{-\infty}^{\infty}$ is not $\lim_{a\to\infty} \int_{-a}^{a}$ but rather if you rewrite the integral that diverges, you can find that the integral converges.
99. (a) $\int_1^{\infty} \frac{1}{x^n}\, dx$ will converge if $n > 1$ and diverge if $n \leq 1$.
(b) 

(c) Converges
101. (a) $\Gamma(1) = 1, \Gamma(2) = 1, \Gamma(3) = 2$ (b) Proof
(c) $\Gamma(n) = (n - 1)!$
103. $1/s,\ s > 0$ **105.** $2/s^3,\ s > 0$ **107.** $s/(s^2 + a^2),\ s > 0$
109. $s/(s^2 - a^2),\ s > |a|$
111. (a) 

(b) About 0.2525
(c) 0.2525; same by symmetry
113. $c = 1$; $\ln(2)$
115. $8\pi[(\ln 2)^2/3 - (\ln 4)/9 + 2/27] \approx 2.01545$
117. $\int_0^1 2 \sin(u^2)\, du$; 0.6278
119. (a) 

(b) Proof

Review Exercises for Chapter 8 (page 591)

1. $\frac{1}{3}(x^2 - 36)^{3/2} + C$ **3.** $\frac{1}{2}\ln|x^2 - 49| + C$
5. $\ln(2) + \frac{1}{2} \approx 1.1931$ **7.** $100 \arcsin(x/10) + C$
9. $\frac{1}{9}e^{3x}(3x - 1) + C$
11. $\frac{1}{13}e^{2x}(2 \sin 3x - 3 \cos 3x) + C$
13. $\frac{2}{15}(x - 1)^{3/2}(3x + 2) + C$
15. $-\frac{1}{2}x^2 \cos 2x + \frac{1}{2}x \sin 2x + \frac{1}{4}\cos 2x + C$
17. $\frac{1}{16}\left[(8x^2 - 1) \arcsin 2x + 2x\sqrt{1 - 4x^2}\right] + C$
19. $\sin(\pi x - 1)[\cos^2(\pi x - 1) + 2]/(3\pi) + C$
21. $\frac{2}{3}[\tan^3(x/2) + 3 \tan(x/2)] + C$ **23.** $\tan\theta + \sec\theta + C$
25. $3\pi/16 + \frac{1}{2} \approx 1.0890$ **27.** $3\sqrt{4 - x^2}/x + C$
29. $\frac{1}{3}(x^2 + 4)^{1/2}(x^2 - 8) + C$ **31.** π